

[arXiv:2201.11901](#) [[pdf](#), [ps](#), [other](#)]

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**Graded extensions of generalized Haagerup categories**

**Authors:** [Pinhas Grossman](#), [Masaki Izumi](#), [Noah Snyder](#)

We classify certain  $\mathbb{Z}_2$ -graded extensions of generalized Haagerup categories in terms of numerical invariants satisfying polynomial equations. In particular, we construct a number of new examples of fusion categories, including:  $\mathbb{Z}_2$ -graded extensions of  $\mathbb{Z}_{\{2n\}}$  generalized Haagerup categories for all  $n \leq 5$ ;  $\mathbb{Z}_2 \times \mathbb{Z}_2$ -graded extensions of the Asaeda-Haagerup categories; and extensions of the  $\mathbb{Z}_2 \times \mathbb{Z}_2$  generalized Haagerup category by its outer automorphism group  $A_4$ . The construction uses endomorphism categories of operator algebras, and in particular, free products of Cuntz algebras with free group  $C^*$ -algebras.

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[arXiv:2108.06561](#) [[pdf](#), [other](#)]

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**Kashaev--Reshetikhin Invariants of Links**

**Authors:** [Kai-Chieh Chen](#), [Calvin McPhail-Snyder](#), [Scott Morrison](#), [Noah Snyder](#)

Kashaev and Reshetikhin previously described a way to define holonomy invariants of knots using quantum  $\mathfrak{sl}_2$  at a root of unity. These are generalized quantum invariants depend both on a knot  $K$  and a representation of the fundamental group of its complement into  $SL_2(\mathbb{C})$ ; equivalently, we can think of  $KR(K)$  as associating to each knot a function on (a slight generalization of) its character variety. In this paper we clarify some details of their construction. In particular, we show that for  $K$  a hyperbolic knot  $KaRe(K)$  can be viewed as a function on the geometric component of the  $A$ -polynomial curve of  $K$ . We compute some examples at a third root of unity.

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[MR4302495](#) **Pending** [Brochier, Adrien](#); [Jordan, David](#); [Safronov, Pavel](#); [Snyder, Noah](#) Invertible braided tensor categories. *Algebr. Geom. Topol.* **21** (2021), no. 4, 2107–2140. [57R56](#)

We prove that a finite braided tensor category  $A$  is invertible in the Morita 4–category  $\mathbf{BrTens}$  of braided tensor categories if and only if it is nondegenerate. This includes the case of semisimple modular tensor categories, but also nonsemisimple examples such as categories of representations of the small quantum group at good roots of unity. Via the cobordism hypothesis,

we obtain new invertible 4-dimensional framed topological field theories, which we regard as a nonsemisimple framed version of the Crane–Yetter–Kauffman invariants, after the Freed–Teleman and Walker constructions in the semisimple case. More generally, we characterize invertibility for  $E_1$ – and  $E_2$ –algebras in an arbitrary symmetric monoidal infinity–category, and we conjecture a similar characterization of invertible  $E_n$ –algebras for any  $n$ . Finally, we propose the Picard group of **BrTens** as a generalization of the Witt group of nondegenerate braided fusion categories, and pose a number of open questions about it.

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[MR4265482](#) [Reviewed](#) [Pollack, Paul](#); [Snyder, Noah](#) A quick route to unique factorization in quadratic orders. *Amer. Math. Monthly* **128** (2021), no. 6, 554–558. [11R11](#) ([11R29](#))

We give a short proof—not relying on ideal classes or the geometry of numbers—of a known criterion for quadratic orders to possess unique factorization.

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[MR4228258](#) [Reviewed](#) [Brochier, Adrien](#); [Jordan, David](#); [Snyder, Noah](#) On dualizability of braided tensor categories. *Compos. Math.* **157** (2021), no. 3, 435–483. (Reviewer: Philsang Yoo) [17B37](#) ([16D90](#) [18M15](#) [57K16](#) [57K31](#))

We study the question of dualizability in higher Morita categories of locally presentable tensor categories and braided tensor categories. Our main results are that the 3-category of rigid tensor categories with enough compact projectives is 2-dualizable, that the 4-category of rigid braided tensor categories with enough compact projectives is 3-dualizable, and that (in characteristic zero) the 4-category of braided multi-fusion categories is 4-dualizable. Via the cobordism hypothesis, this produces respectively two-, three- and four-dimensional framed local topological field theories. In particular, we produce a framed three-dimensional local topological field theory attached to the category of representations of a quantum group at any value of  $q$ .

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[MR4254952](#) [Reviewed](#) [Douglas, Christopher L.](#); [Schommer-Pries, Christopher](#); [Snyder, Noah](#) Dualizable tensor categories. *Mem. Amer. Math. Soc.* **268** (2020), no. 1308, vii+88 pp. ISBN: 978-1-4704-4361-0; 978-1-4704-6347-2 (Reviewer: Jirí Rosický) [57R56](#) ([16D90](#) [17B37](#) [18M15](#) [55U30](#) [57K35](#) [57R15](#))

We investigate the relationship between the algebra of tensor categories and the topology of framed 3-manifolds. On the one hand, tensor categories with certain algebraic properties determine topological invariants. We prove that fusion categories of nonzero global dimension are 3-dualizable, and therefore provide 3-dimensional 3-framed local field theories. We also show that all finite tensor categories are 2-dualizable, and yield categorified 2-dimensional 3-framed local field theories. On the other hand, topological properties of 3-framed manifolds determine algebraic equations among functors of tensor categories. We show that the 1-dimensional loop bordism, which exhibits a single full rotation, acts as the double dual autofunctor of a tensor category. We prove that the 2-dimensional belt-trick bordism, which unravels a double rotation, operates on any finite tensor category, and therefore supplies a trivialization of the quadruple dual. This approach produces a quadruple-dual theorem for

suitably dualizable objects in any symmetric monoidal 3-category. There is furthermore a correspondence between algebraic structures on tensor categories and homotopy fixed point structures, which in turn provide structured field theories; we describe the expected connection between pivotal tensor categories and combed fixed point structures, and between spherical tensor categories and oriented fixed point structures.

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[MR3934626](#) [Reviewed](#) [Douglas, Christopher L.](#); [Schommer-Pries, Christopher](#); [Snyder, Noah](#) The balanced tensor product of module categories. *Kyoto J. Math.* **59** (2019), no. 1, 167–179. (Reviewer: Luz Adriana Mejía Castaño) [18D10](#) ([13C60](#))

The balanced tensor product  $M \otimes_A N$  of two modules over an algebra  $A$  is the vector space corepresenting  $A$ -balanced bilinear maps out of the product  $M \times N$ . The balanced tensor product  $M \boxtimes_C N$  of two module categories over a monoidal linear category  $C$  is the linear category corepresenting  $C$ -balanced right-exact bilinear functors out of the product category  $M \times N$ . We show that the balanced tensor product can be realized as a category of bimodule objects in  $C$ , provided the monoidal linear category is finite and rigid.

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[arXiv:1810.06076](#) [[pdf](#), [other](#)]

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**The Extended Haagerup fusion categories**

**Authors:** [Pinhas Grossman](#), [Scott Morrison](#), [David Penneys](#), [Emily Peters](#), [Noah Snyder](#)

In this paper we construct two new fusion categories and many new subfactors related to the exceptional Extended Haagerup subfactor.

The Extended Haagerup subfactor has two even parts EH1 and EH2. These fusion categories are mysterious and are the only known fusion categories which appear to be unrelated to finite groups, quantum groups, or Izumi quadratic categories. One key technique which has previously revealed hidden structure in fusion categories is to study all other fusion categories in the Morita equivalence class, and hope that one of the others is easier to understand. In this paper we show that there are exactly four categories (EH1, EH2, EH3, EH4) in the Morita equivalence class of Extended Haagerup, and that there is a unique Morita equivalence between each pair. The existence of EH3 and EH4 gives a number of interesting new subfactors. Neither EH3 nor EH4 appears to be easier to understand than the Extended Haagerup subfactor, providing further evidence that Extended Haagerup does not come from known constructions. We also find several interesting intermediate subfactor lattices related to Extended Haagerup.

The method we use to construct EH3 and EH4 is interesting in its own right and gives a general computational recipe for constructing fusion categories in the Morita equivalence class of a subfactor. We show that pivotal module  $C^*$  categories over a given subfactor correspond exactly to realizations of that subfactor planar algebra as a planar subalgebra of a graph planar algebra. This allows us to construct EH3 and EH4 by realizing the Extended Haagerup subfactor planar algebra inside the graph planar algebras of two new graphs. This technique also answers a

long-standing question of Jones: which graph planar algebras contain a given subfactor planar algebra?

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[MR3644793](#) **Reviewed** [Elias, Ben](#); [Snyder, Noah](#); [Williamson, Geordie](#) On cubes of Frobenius extensions. *Representation theory—current trends and perspectives*, 171–186, [EMS Ser. Congr. Rep.](#), *Eur. Math. Soc., Zürich*, 2017. (Reviewer: Jörg Feldvoss) [19A22](#) ([17B10](#) [18A40](#) [18D05](#))

Given a hypercube of Frobenius extensions between commutative algebras, we provide a diagrammatic description of some natural transformations between compositions of induction and restriction functors, in terms of colored transversely-intersecting planar 1-manifolds. The relations arise in the first and third authors' work on (singular) Soergel bimodules.

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