1. Compute
\[ \int_S \text{curl} \, F \cdot N \, dA , \]
where \( S \) is that part of the surface \( y = x^2 + z^2 \) in \( \mathbb{R}^3 \) for which \( 0 \leq y \leq 1 \), \( N \) is the unit normal to \( S \) pointing toward the \( y \)-axis, \( dA \) is the area element, and \( F : \mathbb{R}^3 \to \mathbb{R}^3 \) is the mapping
\[ F(x, y, z) = e^{x^2+z^2}(z, y, -x) \].

2. Suppose that \( f : [0, \infty) \to \mathbb{R}^1 \) is a nonnegative, uniformly continuous function and that
\[ \int_0^\infty f(x) \, dx < \infty . \]
Prove that \( \lim_{x \to \infty} f(x) = 0 \).

3. Let \( f : \mathbb{R}^2 \to \mathbb{R}^1 \) be continuous, and define
\[ g(y) = \int_0^1 f(x, y) \, dx . \]
Assume that \( \frac{\partial f}{\partial y} \) is continuous on \( \mathbb{R}^2 \), and compute \( g'(y) \). Prove your result.

4. The function \( \frac{1}{32} x^4 + x^2 y^2 - x^3 - y^3 - xy^3 \) has critical points at \((24, 0)\) and \((0, 0)\). By a careful analysis, determine whether each point is a local maximum, local minimum or a point which is neither a local maximum nor a local minimum.

5. Let \( f : B \to \mathbb{R}^1 \) be a uniformly continuous function, where \( B \subset \mathbb{R}^n \) is an open ball. Prove that there is a uniformly continuous function \( F \) defined on the closure of \( B \) such that \( F \) restricted to \( B \) is equal to \( f \).
6. Let \( g: \mathbb{R}^2 \to \mathbb{R}^2 \) be defined for points \( x = (x, y) \) by
\[
g(x, y) = (x^2 + y^2 - |x^2 - y^2|, x^2 + y^2 + |x^2 - y^2|).
\]
(a) Give the definition of the differential of \( g \) at \( x_0 \), denoted by \( dg(x_0) \).
(b) Determine those points \( x_0 \in \mathbb{R}^2 \) where \( dg(x_0) \) exists and where it does not exist. In both cases, justify your answer. Be sure to analyze the case \( x_0 = 0 \).
(c) Find those points \( x_0 \in \mathbb{R}^2 \) where \( g \) locally has a differentiable inverse and where it does not. In both cases, justify your answer.

7. Let \( n \) be an integer greater than 1, and consider the following statement: If \( \omega \) is a differential 2-form on \( \mathbb{R}^n \) with the property that \( \omega \wedge \lambda = 0 \) for every differential 1-form \( \lambda \), then \( \omega \) must be the zero form. For what \( n \) is the above statement true? For what \( n \) is it false? Prove your answers.

8. (a) Let \( F: \mathbb{R}^3 \to \mathbb{R}^2 \) be defined by
\[
F(x, y, z) = (x^2 + xy - 1, z^2 + x^2 - y^2 - 2)
\]
and observe that \( F(a) = (0, 0) \) where \( a = (-1, 0, 1) \). Prove that there exist an open interval \( (a, b) \), a \( C^1 \) curve of the form \( \gamma(t) = (f(t), g(t), h(t)) \) with \( a < t < b \), and an open set \( U \subset \mathbb{R}^3 \) containing \( a \) such that
\[
U \cap F^{-1}(0, 0) = \{ \gamma(t) : a < t < b \}
\]
(b) Compute \( \gamma'(t_0) = (f'(t_0), g'(t_0), h'(t_0)) \) where \( \gamma(t_0) = a \).

9. Let \( f: [0, 1] \to \mathbb{R}^1 \) be defined by
\[
f(x) = \begin{cases} 
\frac{1}{k} & \text{if } \frac{1}{k+1} < x \leq \frac{1}{k}, \ k = 1, 2, \ldots \\
0 & \text{for } x = 0 
\end{cases}
\]
(a) For any given \( \varepsilon > 0 \), show how to construct a partition \( P \) of the interval \([0, 1]\) such that
\[
U(P, f) - L(P, f) < \varepsilon.
\]
(\( U(P, f) \) and \( L(P, f) \) are the upper and lower Riemann sums for \( f \) over the partition \( P \)).
(b) Find an expression for
\[
\int_0^1 f(x) \, dx
\]
and justify your answer.