

Algebra Tier 1 Exam – Syllabus¹

Linear Algebra, including

- (1) Vector spaces, linear independence, bases, dimension
- (2) Linear transformations, matrices, change of basis, eigenvalues and eigenvectors, diagonalization
- (3) Spectral theorem for self-adjoint and normal linear transformations on finite-dimensional inner product spaces
- (4) Jordan canonical form

Group Theory, including

- (1) Subgroups, normal subgroups, quotient groups, homomorphism theorems
- (2) Lagrange's Theorem, Cauchy's theorem (on the existence of elements of order p when the prime p divides the group order)
- (3) Standard examples (cyclic groups, symmetric groups)
- (4) Direct products, structure of finite abelian groups

Rings and Fields, including

- (1) Ideals, quotient rings, homomorphism theorems
- (2) Unique factorization domains, principal ideal domains, Euclidean domains, polynomial rings
- (3) Maximal ideals, prime fields, fields, integral domains
- (4) Field extensions, algebraic elements, finite fields

Textbooks. Each of the following textbooks covers the material required for the examination:

Michael Artin. *Algebra*. Prentice Hall, Inc., Englewood Cliffs, NJ, 1991.

David S. Dummit and Richard M. Foote. *Abstract algebra*. John Wiley & Sons, Inc., Hoboken, NJ, third edition, 2004.

I. N. Herstein. *Abstract algebra*. Prentice Hall, Inc., Upper Saddle River, NJ, third edition, 1996. With a preface by Barbara Cortzen and David J. Winter.

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