Cauchy's Integral Formula for Clifford Valued Functions

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1 Description:

This is a project for ambitious students who find Cauchy's integral formula

$$f(z_0) = \frac{1}{2\pi i} \oint \frac{f(z)}{z - z_0}\, dz \tag{1}$$

for complex holomorphic functions $f(z)$ exciting.

Many results of complex analysis have analogues in quaternionic analysis. In particular, there are analogues of complex holomorphic functions called (left and right) regular functions. And there is an analogue of (1) for those functions usually referred to as Cauchy-Fueter formula [7, 2].

Complex numbers $\mathbb{C}$ and quaternions $\mathbb{H}$ are special cases of Clifford algebras [4]. There is a further extension of complex and quaternionic analysis called Clifford analysis. For certain types of Clifford algebras, Clifford analysis is very similar to complex and quaternionic analysis [1, 3, 5, 6]. (Some modifications are necessary due to the fact that quaternions and most Clifford algebras are not commutative.)

The goal of this project is to investigate Clifford analogues of (1) for the remaining Clifford algebras. This project can possibly produce new mathematical results publishable in a research journal.

2 Prerequisites:

Strong background in undergraduate complex analysis, a course on algebra involving rings and modules (needed to understand the definitions of Clifford algebras and modules), some familiarity with integrating differential forms over manifolds and Stokes' Theorem (since we are looking for an integral formula where the loop in (1) is replaced by a higher dimensional contour of integration).

Be aware that the project involves non-commutative setting which may take some time to get used to.

References

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