

Reducing Noise in a Chaotic Signal

an REU project mentored by Mike Jolly

The Lorenz system of differential equations

$$\frac{dx}{dt} = -\sigma x - \sigma y, \quad \frac{dy}{dt} = -y - xz, \quad \frac{dz}{dt} = -bz - br + xy \quad (1)$$

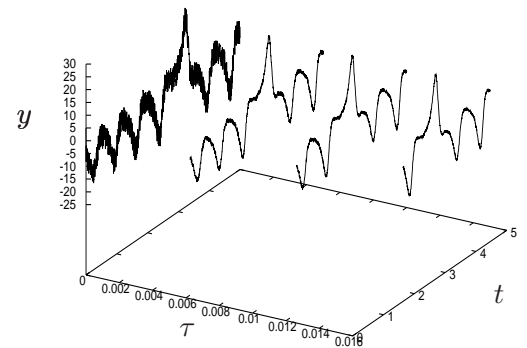
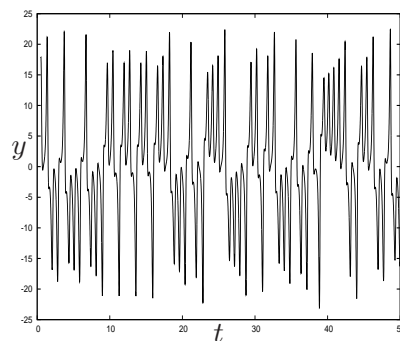
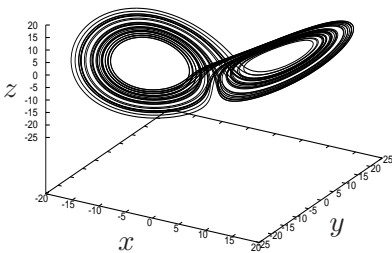
launched the modern theory of chaos. Its *attractor* \mathcal{A} for fixed values of the parameters $\sigma = 10$, $b = 8/3$ and $r = 28$ is shown in the figure below on the left. This particular solution traces out a trajectory which is approached by nearly all other solutions as $t \rightarrow \infty$. The precise nature of that approach, however, is highly sensitive to the initial condition, as components of this particular trajectory oscillate in an irregular manner (as in the middle figure).

Now consider a very different type of differential equation

$$\frac{dv}{d\tau} = -\gamma \sup_t |v - PW(v)|(v - PW(v)), \quad v = v(\tau, t). \quad (2)$$

The phase space of (2) consists of continuous, bounded functions of t , rather than \mathbb{R}^3 as in (1). The function W takes v to a trajectory in \mathbb{R}^3 such that if v is the y -component of \mathcal{A} , then $W(v)$ is (x, y, z) , i.e., all three components of \mathcal{A} . Here γ is an auxiliary damping parameter, and P is the projection onto the y -component. Thus, steady states of (2) (for which $dv/d\tau = 0$) correspond to \mathcal{A} .

The question is, what happens when we start with an initial function $v(0, t)$, which is not the y -component of \mathcal{A} ? It can be useful if, as τ increases, $v(\tau, t)$ approaches y -component of \mathcal{A} . For instance, we can start with $v(0, t)$, $t \in I_0$ being a noisy perturbation of the y -component of \mathcal{A} , over some interval I_0 and evolve (2) to recover a portion of the function $y(t)$, $t \in I_1 \subset I_0$. The preliminary result in the figure on the right indicates that this is possible.



This project is an exploration of how well (2) (and variations on it) can reduce noise in a chaotic signal such as that coming from the Lorenz equations. A student who is likely to enjoy this project is one who has had a course in differential equations, and if not some exposure to a programming language, be willing to learn one from the mentor during the REU program. The description above is meant to give only the flavor of the project, not necessarily be self-contained. The mentor will fill in the details for the student during the REU program.