2015 Indiana Undergraduate Mathematics Research Conference
Titles and Abstracts.

1. Tony Allen [West Virginia U.], Emily Gebhardt [Mercyhurst U.] and Adam Kluball [Bethany Lutheran College] (advisor: Tiffany Kolba, Valparaiso), Noise-Induced Stabilization of Stochastic Differential Equations.

   The phenomenon of noise-induced stabilization occurs when an unstable deterministic system of ordinary differential equations is stabilized by the addition of randomness into the system. Noise-induced stabilization is quite an intriguing and surprising phenomenon as one's first intuition is often that noise will only serve to further destabilize the system. In this presentation, we investigate stochastic differential equations (SDE's) of the form, $dX(t) = b(X(t))dt + s(X(t))dB(t)$, where $b(x)$ is the deterministic drift coefficient, $s(x)$ is the noise coefficient, and $B(t)$ is Brownian motion. We find under what conditions these SDE's are stable, where we take the notion of stability to be that of global stochastic boundedness. Specifically, we find the minimum amount of noise necessary for noise-induced stabilization to occur when the drift and noise coefficients are power, exponential, or logarithmic functions.

2. Leonardo Azopardo [Purdue], Maxim Millan [Purdue], and Sarah Thomaz [Purdue] (advisors: Edray Goins and Avi Steiner, Purdue), Visualizing Dessins d'Enfants on the Torus.

   A Belyĭ map $\beta : \mathbb{P}^1(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$ is a rational function with at most three critical values; we may assume these values are $\{0, 1, \infty\}$. A Dessin d'Enfant is a planar bipartite graph obtained by considering the preimage of a path between two of these critical values, usually taken to be the line segment from 0 to 1. Such graphs can be drawn on the sphere by composing with stereographic projection: $\beta^{-1}\big([0,1]\big) \subseteq \mathbb{P}^1(\mathbb{C}) \simeq S^2(\mathbb{R})$. Replacing $\mathbb{P}^1$ with an elliptic curve E, there is a similar definition of a Belyĭ map $\beta : E(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$. The corresponding Dessin d'Enfant can be drawn on the torus by composing with an elliptic logarithm: $\beta^{-1}\big([0,1]\big) \subseteq E(\mathbb{C}) \simeq \mathbb{T}^2(\mathbb{R})$.

   In this project, we use the open source `Sage` to write code which takes an elliptic curve E and a Belyĭ map β to return a Dessin d'Enfant on the torus – both in two and three dimensions. Following a 2013 paper by Cremona and Thongjunthug, we make the elliptic logarithm $E(\mathbb{C}) \simeq \mathbb{C}/\Lambda$ explicit using a modification of the arithmetic-geometric mean, then compose with a canonical one-to-one correspondences $\mathbb{C}/\Lambda \simeq \mathbb{T}^2(\mathbb{R})$. Using this, we focus on several examples of Belyĭ maps which appear on Elkies' Harvard web page entitled "Elliptic Curves in Nature."

3. Erin Boggess [Simpson College], Jordan Collignon [California State U. Monterey Bay], and Alanna Riederer [U. of Central Oklahoma] (advisor: Alex Capaldi, Valparaiso), An Agent-Based Model Simulating a Reintroduction of the Extinct Passenger Pigeon Under Varying Resource Availability.

   The Passenger Pigeon (*Ectopistes migratorius*) was an iconic species of bird in eastern North America that comprised 25-40% of North American avifauna. Passenger Pigeons went extinct in 1914 due to excessive hunting over the previous 50 years. Current research aims to de-extinct the Passenger Pigeon and someday release the species into its historic range. To determine under which conditions a Passenger Pigeon could survive a reintroduction into a natural habitat, we developed a spatially explicit, agent-based model to simulate the population dynamics of the Passenger Pigeon in a number of present-day forest environments. The model incorporates the following stochastic processes: varying availability of food sources, reproduction, and natural death of the Passenger Pigeon. Bio-energetics, tree distributions and other ecological values were obtained from literature. Results from our simulations suggest that the Passenger Pigeon could survive a reintroduction into a natural environment.

4. Ben Briggs [IUB] (advisor: Chris Connell, IUB), Embedding Everywhere Negatively Curved Surfaces in a Ball.

   We consider the problem of embedding an everywhere negatively curved surface in $B(0,1) \subset \mathbb{R}^3$.By modifying Rozendorn's non-positively curved surface we reduce the problem into two parts: The first an index manipulation aimed at satisfying Poincaré-Hopf topological constraints, and the second a local embedding problem of patches. The first part having been resolved, the patch embeddibility problem amounts to finding solutions for a constrained Codazzi-Mainardi PDE equation with boundary values.

5. Sarah Butchko [IUB] (advisor: Kevin Pilgrim, IUB), Classification of Pillowcase Covers.

   We begin with a surface obtained by gluing two squares together. One square is white and labeled with a Q, the other is black and has the reflection of Q as a label, which we will call Q-bar. This surface Z is called the square pillowcase and is homeomorphic to the sphere. We then can examine covering spaces of the square pillowcase, which are the gluing X of several white and black squares such that each white edge is paired with exactly one black edge. The covering map $f : X \to Z$ from a pillowcase cover to the square pillowcase is done in the obvious way by sending white squares in the cover to the white square labeled Q in the square pillowcase, similarly for the black squares. This research project involves the classification of such pillowcase covers $f : X \to Z$. Classification can be determined in an algebraic way by using permutations that represent each pillowcase cover. This is inherently an idea of sameness between some pillowcase covers and thus we can find equivalency classes in both a visual representation as well as an algebraic process.

6. Daniel Crane [Taylor] and Tanner Dye [Taylor] (advisor: Jeremy Case, Taylor), Counting Gift Exchanges and Comparing their Probabilities in Secret Santa.

   We explore the Secret Santa gift exchange problem. A group of n people draws names at random, giving a gift to the person drawn. Generalizing the well-known Hat-Check problem, we examine the probabilities of gift exchanges under various scenarios. We then consider the probabilities of certain gift exchanges when people take turns drawing names and develop a strategy to maximize the likelihood of receiving a gift from the most generous participant.

7. Marika Diepenbroek [University of North Dakota], Monica Maus [Minnesota State University Moorhead], Alex Stoll [Clemson University] (advisor: Lara Pudwell, Valparaiso), Pattern Avoidance in Reverse Double Lists.

   Pattern avoidance is a branch of combinatorics that arose in 1968 when Donald Knuth began studying stack sorting. One central problem in pattern avoidance is finding the number of permutations of length n that avoid a specific pattern ρ. We expanded this problem to reverse double lists, or lists built by combining a permutation with its reverse. We computed the number of reverse double lists of each length that avoid patterns of up to length four and then conjectured and proved formulas to explain these sequences. In this presentation, we will specifically focus on avoiding the pattern 1342.

8. Morgan Escalera [Rose-Hulman] (advisor: Wayne Tarrant, Rose-Hulman), What it means to be Too Big to Fail.

   In the wake of the 2008 financial crisis that began in the United States but expanded worldwide, the FSB (Financial Stability Board) and the BCBS (Basel Committee on Banking Supervision) created a list of Globally Systematically Important Banks, and suggested that jurisdictional bodies regulate these financial giants more stringently. The methodology for their determination was a compilation of 5 indicators: size, interconnectedness, cross-jurisdictional activities, complexity, and substitutability. It is the purpose of this research paper to slightly modify those 5 criteria and apply it to governmental bodies, using the case of the troubled economy of Greece in the Eurozone as the primary example. The goal will be to apply this to other countries.

9. Christopher Felder [Butler] (advisors: Carl Cowen, IUPUI, Eva Gallardo, U. Complutense de Madrid and Rebecca Wahl, Butler), An Investigation of the Berezin Transform of Composition Operators.

   Properties of an operator are sometimes revealed by the Berezin transform of the operator. The Berezin transform also associates very complicated operators with a more simple object a subset of the complex plane. We investigate, with the help of Matlab, several examples of the Berezin transform of a composition operator on the Hardy space. One of these examples, with a particularly interesting geometry, is given special consideration.

10. Sharon Freshour [IUPUI and St. Edwards U.] (advisors: Yaroslav Molkov, Emily Yoo, and William Barnett, IUPUI), Modeling the Firing Rate Pattern of the Vagus Nerve.

    The process of breathing, also known as respiration, is composed of a period of inspiration and a period of expiration. During inspiration, which is an active process, inhalation is facilitated by phrenic nerve activity instructing the diaphragm to contract, allowing the lungs to expand and fill with air. In the Vagus nerve, the firing rate pattern seen during inspiration corresponds to the motor output of the phrenic nerve. During expiration, which is a passive process, exhalation is facilitated by inactivation of the phrenic nerve, leading the diaphragm to relax to its resting state and the lungs to expel air. However, unlike the Vagal firing rate pattern seen during inspiration, the Vagal firing rate pattern seen during expiration does not correspond to the motor output of the phrenic nerve. While the phrenic nerve is silent during expiration, the firing rate pattern of the Vagus nerve shows slowly decreasing activity during expiration. Thus, in an attempt to better understand the motor functions responsible for the Vagal activity seen during expiration, the goal of this project is to create a model for the Vagus nerve firing rate pattern that may provide insight into the motor outputs of expiration.

11. Seth Hamilton [Valparaiso] and Breanna Struss [Valparaiso] (advisor: James Caristi, Valparaiso), Designing a Self-reversing Track Layout With TrackMaster$^{\text{TM}}$ Tracks.

    This research project sought to find general track formations that allowed battery operated locomotives to traverse the entire train track in both directions infinitely. These formations allowed for any number of track pieces from TrackMaster$^{\text{TM}}$ Thomas and Friends$^{\text{TM}}$ sets by Fisher Price®. The research looked at start position of the train and presetting of the switches as well as what is necessary to have a complete track with no dead ends. Surprisingly, there was found to be only one track formation that allowed for entire traversal of the track in both directions. This layout was termed a “dog bone” and consisted of two switch pieces connected at the ends with the tips of both switches connecting to themselves on the same switch. A proof that this layout is the only layout that satisfies the conditions is given.

12. Andrew Henderson [IUB] (advisor: Dylan Thurston, IUB), Developing an algorithm for the treatment of elastic networks.

    In this project, elastic networks were defined as undirected graphs with weighted edges (rubber bands tied together are good examples of these networks). Stretching an elastic network in $\mathbb{R}^1$ can be thought of as being equivalent to an electrical network and can be examined using the techniques of computing the equivalent resistance across a network of resistors. The first problem this project approached was a Dirichlet problem involving an L-shaped graph.

    Secondly, this project developed methods for embedding elastic networks inside “pipe graphs” which were defined as undirected graphs with edge weightings corresponding to lengths of pipes. In this case, the use of electrical methods breaks down as Kirchhoff’s laws no longer apply. Thus, an algorithm was developed to numerically compute the energies of such embeddings.

13. Manasse Kota Kwete [IUB] (advisor: Kevin Pilgrim, IUB),The graph-approximation for the Dirichlet problem with "resistors" of variable resistance R in an "electrical network."

Let D denote an open bounded connected region in the plane the unit disk, and let D' denote its closure. Dirichlets Problem is: given a bounded, piecewise continuous function on the boundary of D, $u : \partial D \to \mathbb{R}$, construct a harmonic function on the disk, $u : D \to \mathbb{R}$ extending the given boundary values. Under very general conditions, the solution exists and is unique. In this paper, we consider a particular Dirichlet problem when D consists of an L shaped union of five congruent squares and the boundary values $u = 0, u = 1$ are chosen in a certain way. It turns out that with our particular choices, as D is stretched horizontally or vertically, the Dirichlet energy $E(u) = \int_D |\nabla u|^2 dA$ stays bounded. Finding u exactly is hard. We consider graph approximations to D, interpret the Dirichlet problem in terms of electrical networks, interpret stretching in terms of varying resistance, and investigate what happens.

14. Dylan Linville [Rose-Hulman] and Daniel Trugillo Martins Fontes [U. of São Paulo] (advisor: Mark Panaggio, Rose-Hulman), Spontaneous Synchrony on Graphs and the Emergence of Order from Disorder.

From pulsars to pedestrians and bacteria to brain cells, objects that exhibit cyclical behavior, called oscillators, are found in a variety of different settings. When oscillators adjust their behavior in response to nearby oscillators, they often achieve a state of synchrony in which they all have the same phase and frequency. Here, we will explore the Kuramoto model, a simple and general model which describes oscillators as dynamical systems on a graph and has been used to study synchronization in systems ranging from firefly swarms to the power grid. We will discuss analytical and numerical methods used to investigate the governing system of differential equations and the conditions that lead to synchronization, and demonstrate that perfect synchronization occurs only under strict conditions and for specific graph structures. We will also present results from an experiment with coupled metronomes in which spontaneous emergence of synchronization, consistent with the mathematical theory, can be observed in a real-world setting.

15. Bronz McDaniels [Purdue], Danny Sweeney [Purdue], Sofia Lyrintzis [IPFW] and Yesid Sanchez Arias [U. Nacional de Colombia] (advisors: Ezra Goins and Hongshan Li, Purdue), Examples of Belyĭ Maps for Elliptic Curves.

A Belyĭ map $\beta : \mathbb{P}^1(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$ is a rational function with at most three critical values; we may assume these values are $\{0, 1, \infty\}$. Replacing $\mathbb{P}^1$ with an elliptic curve $E: \ y^2 = x^3 + A\,x + B$, there is a similar definition of a Belyĭ map $\beta : E(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$.

This project seeks to determine examples of Belyĭ maps for elliptic curves. We have shown that given any elliptic curve E there exist infinitely many Belyĭ maps of degree 2; they are in the form $\beta(x, y) = (a\,x + b)/(c\,x + d)$. We have also shown that any elliptic curve has at least one Belyĭ map of degree 3 with critical values $\{0, 1, \infty\}$. After placing the curve in Hessian normal form $y^2 + a_1\,x\,y + a_3\,y = x^3$, the Belyĭ map is in the form

$$\beta(x, y) = \frac{\left(2\,a_1^3 - 27\,a_3 + 2\,\sqrt{a_1^6 - 27\,a_1^3\,a_3}\right)y - 27\,a_3^2}{\left(2\,a_1^3 - 27\,a_3 - 2\,\sqrt{a_1^6 - 27\,a_1^3\,a_3}\right)y - 27\,a_3^2} \qquad \text{for } a_1 \neq 0.$$

16. Sam Pilgrim [IUB] (advisor: Jeffrey Meier, IUB), Two and Three Bridge Trivial Tangles with Closures to the Unknot.

    We investigate subgroups of the braid group that act as stabilizers on two particular n-stranded tangles. The Wicket Group, W_n, gives the stabilizers for the first of the two tangles, which is simply n wickets, and the stabilizers of the other tangle can be found by conjugating the generators of W_n. We will give presentations for the intersection of these two subgroups of B_{2n} for the cases of $n = 2$ and $n = 3$. These groups, together with the classification of $(2, 1)$ and $(3, 1)$ bridge trisections, gives a complete list of two- and three-bridge trivial tangles whose union with either of our two original tangles is the unknot. Consequently, this process also classifies all four and six stranded braids whose plat and shifted plat closures are both the unknot. Since this result relies on the classification of knotted surfaces in $\mathbb{R}^4$, there will be a brief discussion of how triples of trivial tangles give rise to these surfaces.

17. Zachary Wampler [IUB] (advisor: Matt Bainbridge, IUB), Veni, vidi, Veech-i: Using the Veech groups of the regular octagon and double pentagon translation surfaces to classify their degree 2 covering spaces.

    We obtain the equivalence classes of degree 2 covers of the regular octagon X_8 and double pentagon X_5 translation surfaces by computing the orbits of the action of the Veech group on their first cohomologies.

18. Thomas Yahl [IUPUI] and Chris Neuffer [IUPUI] (advisor: Carl Cohen, IUPUI), Linear Fractional Composition Operators in Two Complex Variables.

    Linear fractional maps and their composition operators have been well studied in one complex variable. As a natural extension of this study, linear fractional maps have been generalized to several complex variables. Our study has been these mappings in two complex variables, where they have been classified into seven equivalence classes by the behavior of their iterates on the open ball and their fixed points in the closed ball. Specifically, we focus on the relationship between the eigenvalues of the matrix representation of these maps with the fixed points of the maps themselves along with the eigenfunctions of compositions of the linear fractional maps with analytic functions mapping the ball into the complex plane.