2018 Indiana Undergraduate Mathematics Research Conference
Titles and Abstracts.

* Prof. Ciprian Demeter [IUB] (Kenote Presentation), Decouplings and applications

I will show how a recently developed Fourier analytic tool impacted the area of exponential sums and Diophantine systems of equalities.

1. Eric Albers [Temple U.] (advisor: Nicholas Miller, IUB), Constructing Congruence Surfaces.

   In this talk we review the basics of hyperbolic geometry and outline the way discrete subgroups of $PSL_2(\mathbb{R})$ can be constructed via quaternion algebras over number fields. We will provide the definition of a congruence subgroup for the discrete subgroup $PSL_2(\mathbb{Z})$ and talk on how this definition generalizes. With the time remaining we introduce a question posed by Reid about the possible genera for congruence surfaces and present the result that there is no congruence surface constructed from a maximal order in a quaternion algebra of genus $g = 212$.

2. Maryam Amran [UCI] (advisor: Jared Barber, IUPUI), Effects of Wall Proximity on Red Blood Cell Interactions.

   Red blood cells play a vital role in transporting oxygen through microvessels. To help fulfil the bodys oxygen needs, blood is composed of 40-45% red blood cells by volume. As blood flows in a vessel, cell-wall interactions push cells away from the vessel wall creating a region devoid of cells near that wall (cell-free layer) while cell-cell interactions push cells towards the wall and limit the size of that cell-free layer. How these cell-wall and cell-cell forces balance determine the size of the cell-free layer which can affect the speed and distribution of blood and its nutrients throughout the body. To understand these cell-cell and cell-wall interactions better, we simulate two-dimensional pairs of cells interacting in simple linear shear flow in a walled channel. Cells can experience dancing, swapping, and passing interactions depending on cells relative positions and orientations as they enter an interaction region. Decreasing the channel width, which corresponds to decreasing average cell distances to walls, changes the relative frequency of these interactions. To better understand the implications of these interactions, we quantify these interactions by measuring the displacements that cells undergo as they interact. We will also share our findings on the dependency of interaction displacements on cell orientation and relative cell position within the channel. While additional analysis needs to be done, these results suggest that wall proximity and cell orientation can have appreciable effects on the cell-free layer and overall blood and nutrient distribution.

3. Elizabeth Anderson [Villanova U.], Jared Ott (U. Nebraska-Lincoln), and Gwyneth Terrett (Taylor U.) (advisor: Daniel Maxin, Valparaiso), Asymmetric Demographic Models with a Mate-Finding Allee Effect.

   In a two-sex demographic model, the most challenging mathematical components are the couple-formation functions. These functions link the number of pairs with the number of available singles. They are usually not detailed enough to include important aspects of social behavior such as: motivation for pairing which may be gender specific, scarcity or abundance of the opposite gender or social/economic factors. In this research we analyze several two-sex models to better describe asymmetric demographic situations. In particular we focus on a mate-finding Allee effect which models the difficulty of pairing at low population densities, and investigate whether this effect is sensitive to changes in sex ratios and/or overall female/male densities. We also compute the Allee threshold which separates population extinction from persistence and test these results against real demographic data from world populations.

4. Leah Andrews [C. of Wooster], Maya Lapp [St. Olaf C.], and Giorgio Raimondi [Johns Hopkins U.] (advisor: Julia Arciero, IUPUI), Modeling Optimal Treatment Strategies for Transplant Patients.

   Solid organ transplantation is a lifesaving procedure that requires lifelong immunosuppression to prevent organ rejection by the host immune system. While this treatment protects the graft, it often leads to complications including hypertension, nephrotoxicity, and cardiovascular disease. Thus, an improved treatment regime is needed to promote long-term graft tolerance and health of the host. Regulatory T cells (Tregs) are a key component of the host immune system that have the capacity to target specific antigens and whose fundamental role is to inhibit inflammation. Thus, increasing host Treg levels through adoptive transfer has been proposed as an alternative method for promoting graft tolerance. This study adapts a previous ODE model of murine heart transplant rejection to include Treg adoptive transfer and immunosuppression via calcineurin inhibitors (CNI) and mechanistic targeting of rapamycin (mTOR). An optimal delivery of Treg adoptive transfer is predicted by the model to extend graft lifetime ten times longer than without treatment. Since adoptive transfer alone cannot prevent eventual graft rejection, combinatorial treatment strategies are explored by varying Treg dose magnitude, timing, and frequency and immunosuppressant class, timing, and frequency to maintain graft survival with minimal immunosuppression. These theoretical results can guide in vivo experiments aimed at promoting graft tolerance while minimizing the complications presented by immunosuppression.

5. Emi Brawley [Stony Brook U.] (advisor: Noah Snyder, IUB), Trivalent Categories over Finite Fields.

   In this paper, we study and classify trivalent categories over finite fields, i.e., planar algebras generated by the Temperley-Lieb planar algebra and a trivalent vertex. Our results are that, where $\mathcal{C}_n$ is the space of diagrams with n boundary points, the trivalent categories with $\dim \mathcal{C}_n$ bounded by 1, 0, 1, 1, 4, 10 for $0 \leq n \leq 5$ are the golden categories, quantum $SO(3)$, a one-parameter family of ABA categories, and quantum G_2; this parallels the classification of trivalent categories over $\mathbb{C}$, but the number of categories in several of these families—in particular, the golden categories and the ABA categories—depends on the base field chosen. We also classify some trivalent categories that arise when the nondegeneracy assumptions are weakened, and give directions for further investigation in this area.

6. Cory Bugelholl [Franklin C.] and Haley Sledge [Asbury U.] (advisor: Jared Barber, IUPUI), Calibration of a two-dimensional model of normal and cancerous breast cells using image and sensitivity analysis.

   Breast cancer affects 3.5 million people in the United States every year. Further, approximately 90% of cancer-related deaths are due to metastasis, the spreading of cancer from a primary to a secondary site. To better understand the spread of cancer through the vascular system as well as in diagnostic microfluidic devices, a two-dimensional computational model of red blood cell motion in microvessels has been adjusted to consider cancer cell motion through viscous fluid (Stokes flow). Key to a predictive and useful model, however, is calibration of that model using experiments. We have constructed tools for this calibration that allow us to analyze how parameter values affect the model (sensitivity analysis), to extract cell from experimental pictures (image analysis), and to compare model results with those shapes. For the sensitivity analysis, we inspect how membrane stretching and bending elasticities and the cell interior elasticity affect model steady state shapes. In addition, edge detection is used to extract cell shapes and optimization procedures are used to convert those experimental shapes into initial shapes that can be used to run comparative simulations. Finally, using standard linear conversions and rotations, model results from the comparative simulations are compared with experimental shapes. While there is still more calibration to be done, these tools should help produce a model that can help us better understand the physical and chemical properties of circulating cancer cells, optimize microfluidic devices, and ultimately discover insights into ways that metastasis can be mitigated.

7. Amy Carpenter [Lee U.] and Allison Torsey [SUNY Buffalo] (advisor: Julia Arciero, IUPUI), Analyzing the Dynamics of an Inflammatory Response to a Bacterial Infection in Rats

   Sepsis is a serious health condition that is not well understood. It is defined as an overactive immune response that causes severe damage to healthy tissue, often resulting in death. Mathematical modeling has emerged as a useful tool to investigate key elements of the immune response and thus offers a useful method for studying sepsis. Here, a system of four ordinary differential equations is developed to simulate the dynamics of bacteria, the pro-inflammatory immune response, anti-inflammatory immune response, and tissue damage. The pro-inflammatory response is triggered by the presence of bacteria and leads to destruction of bacteria as well as damage to the tissue once the level of inflammation exceeds a certain threshold. The anti-inflammatory response works to temper the pro-inflammatory response, although it is not always capable of preventing sustained tissue damage. The model is used to assess the conditions under which health, aseptic (inflammation-driven) death, or septic (bacteria-driven) death is predicted in both the presence and absence of an induced E. Coli bacterial infection in rats. Model parameters are fit to experimental data from rat sepsis studies. The model is used to predict the survivability range for an infection while varying the initial amount, growth rate, or virulence of the bacteria in the system.

8. Giana Cirulli [Eastern U.], Ezequiel Estrada [Pomona C.], and Sierra Knavel [Ohio U.] (advisor: Jon Beagley, Valparaiso), Constructing Copoint Graphs of Convex Geometries.

   We work with copoint graphs of convex geometries. Copoint graphs can be used to study the complex and fairly recent field of convex geometries. Comparing copoint graphs and their convex geometries helps identify properties. We demonstrate that multiple convex geometries have the same underlying copoint graph. All graphs on one to five vertices can be represented as possible copoint graphs of some convex geometry. Furthermore, we construct several infinite classes of copoint graphs including the complete k-partite graph, path graph, centipede graph, ladder graph, comb graph, pom-pom graph, shark teeth graph, and broken wheel graph.

9. Cornell Holmes [Johns Hopkins U.] (advisors: Chris Connell, IUB), An Investigation of the Berezin Transform of Composition Operators.

   In the paper "Simplices of maximal volume in hyperbolic n-space," U. Haagerup and H. Munkholm estimate the volume of the ideal regular hyperbolic n-simplex by relating it to the volume of the regular euclidean n-simplex inscribed in the unit sphere. In this paper, we use analogous techniques and the Lasserre-Avrachenkov theorem to generate estimates for the volumes of ideal hyperbolic n-simplices by comparing the volumes of ideal hyperbolic n-simplices to the volumes of the euclidean n-simplices with the same vertices as the hyperbolic simplices in the projective model.

10. Malachi Jones [Colgate U.], Victoria Knutson [St. Olaf C.], and Benjamin Stockton [U. Wisconsin-Whitewater] (advisor: Hui Gong, Valparaiso), Optimizing the Creditworthiness Threshold of a Bivariate Distribution.

    Financial institutions must evaluate credit applications when deciding to issue credit. Creditworthiness varies amongst the applicants. Creditors must decide which applications to accept in order to maximize profit. For this paper, we assume applicants are divided into Good and Bad populations. We found optimal threshold values that maximized the creditor's profit under varying assumptions of Normal, χ^2, and Γ- distributions. To do so, we optimized the profit function with respect to the threshold value and we ran simulations to find the threshold value that maximizes the profit.

11. Sophia Kardadi [U. of Notre Dame] and Sarah Pugliese [Brown U.] (advisor: Andres Tovar, IUPUI), Investigating Advanced Topology Optimization Methods.

    Topology optimization is a numerical method to find the optimal distribution of a given amount of material that maximizes the performance of the resulting structure, which is subjected to boundary conditions that include external forces and heat loads. An effective approach to solve a topology optimization problem is the use of the level set method. In this method, the boundary of an n-dimensional structure is defined as the zero level set map of a (n+1)-dimensional surface. Benefits of the level set method include an easily adaptive topology, the ability to set parameters that change complexity of the resulting object, and speed of the code. Drawbacks include intermittent re-initialization of the level set function and ill-posed, steady state solutions. This work studies various implementations of the level set method and compare them to traditional density-based methods, which are widely used in topology optimization. Phase field methods are traditionally used to solve phase separation problems in physical chemistry, and recently there has been a growing interest in applying these methods to structural topology optimization. This article provides a description of the conceptual basis of phase field methods and the equations used to solve the minimum compliance problem in topology optimization. A specific equation and finite element-based solution procedure introduced by Takezawa and co-authors in 2010 is used in this work to solve a minimum compliance problem on a two-dimensional design domain. The scheme introduces a double-well potential function into the objective functional in order to penalize intermediate densities. The description of this procedure includes a discussion of the influence of parameter values on the result. Finally, the phase field method is compared to the more common density-based method, taking into account the computational efficiency as well as the properties of the solution.

12. Julia Krull [Millikin U.], Eric Redmon [Lewis U.], and Andrew Reimer-Berg [Eastern Mennonite U.] (advisor: Lara Pudwell, Valparaiso), Pattern Packing in Words.

    A word is an ordered list of numbers. Specifically, a permutation is a word without repeated letters, denoted π. A pattern is a word we look for within other words, denoted with ρ. In general, permutations are studied in terms of pattern avoidance, that is, which words avoid which patterns. Researchers have discovered several orderly ways to count pattern avoiding words of the form $\pi\pi$, i.e., a permutation followed by itself, and $\pi\pi^r$, i.e., a permutation followed by its reverse. Instead of avoiding patterns, we study pattern packing; that is, we identify words with as many copies of a pattern as possible. Our focus is packing patterns in words of the aforementioned structures. In particular, given a pattern ρ, we consider how many times we can pack ρ into words of these forms, what the ρ-optimal words look like, and how many ρ-optimal words exist for a given length of π.

13. Qiuyun Li [IUB] (advisor: Manda Riehl, Rose-Hulman), Identifying Helical Patterns in Quorum-Regulatory RNA using Boltzmann sampling.

    RNA (ribonucleic acid) is a single-stranded molecule that contains linked nucleotides which pair via hydrogen bonds, forming an object folded on itself. The sequence of nucleotides is its primary structure, while the way it folds on itself is its secondary structure. The secondary structure is crucial to its function, and predicting how a primary structure will fold into a secondary structure is in general a complicated question. Rogers and Heitsch used Boltzmann sampling of possible secondary structures followed by helix similarity analysis to uncover which features are most prevalent in a particular RNAs secondary structures. We restrict our investigation to quorum-sensing RNAs in bacteria, those RNAs whose function is to activate and inhibit various genes in response to their population density. We will present evidence that some quorum-sensing RNAs are stable under this Boltzmann sampling, in that the number of featured helix classes does not change, while others are more unstable.

14. Adam Lonnberg [U. Evansville] (advisor: Alexey Kuznetsov, IUPUI), Fear acquisition and extinction in a new amygdala model.

    The amygdala is involved in the acquisition and extinction of the fear response. Efforts to replicate the results of an existing model (Alexandre & Carrere Front. Syst. Neurosci. 9:41) have obviated several improvements that could be made therein. The new model follows classical rate model formalism and is conditioned to yield robust physiologically relevant activity. The model reproduces antagonistic inhibition between sub-regions of the lateral subdivision of the central nucleus (CeLOn and CeLOff) and basal amygdala (BAf and BAe), as well as the lateral amygdala. Thus, a simplified model, excluding transmembrane potentials as variables, has been proposed. Our model implements Hebbian learning to provide synaptic potentiation responsible for acquisition and extinction. The model also includes acetylcholine, which acts as a modulator of synaptic connections in BA that allows for extinction. The model provides mechanistic connection between the structure of the amygdala and its function in fear acquisition and extinction. The future goal is to implement drug-induced modulations of the model parameters and make predictions on their influence on amygdaloid functions.

15. Amish Mishra [Taylor U.] and Maddison Guillaume [Taylor U.] (advisors: Derek Thompson, Taylor), Numerical Range of Toeplitz Matrices over Finite Fields.

    In this paper, we characterize the k-th numerical range of all $n \times n$ Toeplitz matrices with a constant main diagonal and another single, non-zero diagonal, where the matrices are over the field $\mathbb{Z}_p[i]$, with p a prime congruent to $3 \mod 4$. We find that for $k \in \mathbb{Z}_p^*$, the k-th numerical range is always equal to $\mathbb{Z}_p[i]$ with the exception of the scaled identity. We also classify the 0-th numerical range of the same matrices. Lastly, we conclude with a conjecture regarding the general numerical range of all triangular Toeplitz matrices.

16. Garrett Mulcahy [Purdue U.] (advisor: Alexey Kuznetsov, IUPUI), Modeling the Basal Ganglia.

    The basal ganglia (BG) are a collection of nuclei located deep beneath the cerebral cortex. These nuclei are especially concerned with the selection of an action and the repression of competing actions; thus, they play a critical role in higher-level behavior dynamics. These behavior dynamics are tested in a task called instrumental conditioning (IC), in which an animal is rewarded for choosing a certain action. In this project, we analyze the mechanism by which the BG enable learning of the rewarded action. We applied a model for the BG developed by Kim et al. (2017) to a standard two-choice IC task. After adjusting parameter values to replicate BG-mediated initial learning and reversal, we began to explore what happens when the BG go awry. Dysfunction of the BG normally results in hypokinetic or hyperkinetic diseases, typical examples of which are Parkinsons Disease and Huntingtons Disease (respectively). Guided by physiological research on the pathology of these two diseases, we then modified our healthy BG model to create disease models for Parkinsons Disease and Huntingtons Disease.

17. Matisse Peppet [MIT] (advisor: Alex Kruckman, IUB), Incidence Structures Beyond Projective Planes.

    In this talk, Ill introduce (m, n)-pseudoplanes as generalizations of abstract projective planes. Well see how the right definition of non-degeneracy allows us to define constants in analogy with the notion of order in projective planes. Using these constants and a counting argument, well show that there are no finite non-degenerate (m, n)-pseudoplanes for m or n greater than 2. Infinite non-degenerate (m, n)-pseudoplanes do indeed exist, though, and the counting arguments yield relationships between the constants which are infinite cardinals in these.

18. Jacob Prinz [U. Maryland] (advisors: Nachiket Karnick, Amr Sabry and Noah Snyder, IUB), Cayley's Theorem for Infinity Groupoids.

    In this talk, I will present a method for discerning the structure of freely generated infinity-groupoids. There is an analogous problem for groups, which is the so called "Word Problem": given a finitely presented group, determine if two words are equal. The method is based on a generalization of Cayley's theorem for groups to infinity-groupoids, and allows the word problem to be solved in specific cases for both groups and infinity groupoids.

19. James Marshall Reber [Purdue U.] (advisor: Graham White, IUB), Markov Chains, Mixing Times, and Couplings.

    Markov chain mixing times have been of great interest in recent times. A classic mixing time result, due to Diaconis, is on how many riffle shuffles are required to shuffle a deck of n cards - the required number is $3/2 * log_2(n)$. In this talk, we explore the mixing times of random walks on various graphs using a combinatorial method called coupling. In particular, we give upper bounds on the mixing times of simple random walks on certain families of three-regular graphs, and conjecture some possible generalizations.

20. Ben Riley [U. Kentucky] (advisor: Carmen Rovi, IUB), Cut-Paste Operations and Bordism of Manifolds in an Equivariant Setting.

    This talk will provide an introduction to the concept of Cut-Paste relations for both oriented and non-oriented manifolds. We will present some examples of Cut-Paste invariants along side introducing generalizations of Cut-Paste relations and their deeper connections to bordism. These notions were first discussed in the work of Karras, Kreck, Neumann, and Ossa. Our main goal will be to expand upon classical results and generalize to an equivariant setting.

21. John R. Rowe [Clemson C.] (advisor: Andres Tovar, IUPUI), Adjoint Methods in Topology Optimization.

    The adjoint method serves as a way to compute the sensitivities of an objective function in optimization problems. This method is applied in the context of topology optimization, which seeks to find the optimal distribution of material that maximizes the performance of the system. In topology optimization problems, there are generally a large number of design variables, which makes the direct method of computing the sensitivities less efficient. The adjoint method is another way to find the sensitivities analytically, such that the number of equations to be solved is independent of the number of design variables. The goal of this research is to understand the benefits of the adjoint method, in comparison with the direct method, and apply it in the context of geometrically nonlinear, structural optimization problems. The results of nonlinear models are compared to those of linear models, in which nonlinear models yield a more optimal solution to the problem.

22. Erin Zhao [IUPUI] (advisor: Jared Barber and Julia Arciero, IUPUI), Modeling acute blood flow responses to a major arterial occlusion.

    Peripheral arterial disease (PAD) is a serious illness in which major arteries become blocked, causing reduced blood flow to peripheral tissues. In some cases, the body can effectively compensate after an arterial occlusion, but often PAD results in tissue death that leads to severe pain and possible limb loss. Improved diagnoses and treatments for PAD require a more complete understanding of the changes that occur in vascular segments both proximal and distal to the site of occlusion; such information is investigated using mathematical modeling. Here, a mechanistic model of the vascular network of the rat hindlimb is developed to predict the immediate (acute) changes in vessel diameters and smooth muscle tone following femoral arterial occlusion. The relative importance of these changes in different vessel types on flow compensation is assessed. The model is calibrated to experimental data in a non-occluded reference state. Vascular responses to changes in pressure (myogenic response), shear stress, and metabolic levels are modeled, and both resting and exercising conditions are simulated and compared with experimental data. Despite significant acute dilation of collateral arteries, model results show that the oxygen saturation and blood flow in the calf following occlusion are still reduced, and the deficits become more significant as activity levels increase. This suggests that long-term (chronic) structural adaptations are also necessary to restore blood flow to healthy, pre-occlusion levels.

23. Fan Zhou [Harvard U.] (advisor: Chris Connell, IUB), Series Expressions for the Volume of Hyperbolic Simplices

    We give a power series in terms of the length matrix for the volume of a simplex in a constant curvature ± 1 space convergent in parts of both the spherical and the hyperbolic case and give a convergence condition. We compare this to a known power series by Aomoto which converges only in the spherical case. We extend our methodology to study the volume of ideal (not necessarily regular) hyperbolic simplices. We also restate a result of Aomoto and Kohno in which the volume of a spherical/hyperbolic simplex is expressed as an iterated integral over the space of Gram matrices, but we do so in terms of the length matrix rather than the Gram matrix. We also give an alternate (and perhaps simpler) proof of Tuynman's formula in which the sine of (half of) the area of a spherical/hyperbolic 2-simplex is related to the square root of the determinant of the length matrix.