**PROJECTS 2025**

**Mentor:** Matvei Libine

**Title:** Conformal Transformations on $\mathbb{R}^{p,q}$ and Clifford-Valued Functions

**Project Description:** This project is motivated by complex analysis, but a course in complex variables is *not* a prerequisite. Complex numbers $\mathbb{C}$ and quaternions $\mathbb{H}$ are special cases of Clifford algebras, and many results of complex analysis have analogues in quaternionic analysis as well as Clifford analysis. The goal of this project is to study symmetry properties of Clifford algebra analogues of differentiable functions – called *holomorphic Cliffordian functions* – under conformal or Möbius transformations.

**Prerequisites:** Strong background in multivariable calculus, linear algebra (including quadratic forms), and a course on algebra involving rings and modules (needed to understand the definitions of Clifford algebras and modules).

Exposure to an introductory course on complex variables would help to understand the motivation behind the project, but is not a prerequisite. The project involves a non-commutative setting, which may take some time to get used to.

**Mentor:** Elif Uskuplu

**Title:** Formalization of Combinatorial Design Theory in Proof Assistants

**Project Description**: Combinatorial design theory is a rich mathematical field that investigates the arrangement of elements into specific patterns under precise constraints. Despite its applicability to cryptography, coding theory, and experimental design, its formalization in proof assistants remains largely unexplored. In this project, students will formalize foundational results in combinatorial design theory, such as block designs, Latin squares, and Steiner systems, using the proof assistant Lean. The project will involve studying key theoretical results and developing structured libraries to verify the correctness of designs. The outcomes can pave the way for future automated reasoning in combinatorial design applications.

**Prerequisites:** Familiarity with basic combinatorics (e.g., permutations, combinations, and graph theory). Basic familiarity with the proof assistant Lean is helpful but not required (I will spend two weeks to teach Lean). Interest in discrete mathematics and formal reasoning.

**Mentor:** Elif Uskuplu

**Title:** Formalizing the Transcendence of $\pi$

**Project Description**: This project focuses on formalizing the proof that $\pi$ is transcendental, based on the Lindemann–Weierstrass theorem. Students will develop tools in Lean for algebra, calculus, and complex analysis, and contribute a formalized proof of $\pi$'s transcendence to Lean's mathlib.

**Prerequisites**: Basic algebra (polynomials, field extensions) and analysis. Interest in formal proofs; Lean experience helpful but not required. (I will spend two weeks to teach Lean)

**Mentor**: Babak Seradjeh

**Title:** Self-Adjoint Extensions of Dirac Operators in Corner Geometries

**Project Description:** Dirac operators play a special role in quantum mechanics on continuum bounded geometries, offering an effective description of fundamental physics and universal properties of quantum materials in such geometries alike. Index theorems relate the existence of bound states of Dirac operators in the presence of topological configurations in the background geometry (such as vortices) to the nontrivial topology of the quantum state in the continuum. The presence of such bound states depends crucially on the boundary conditions, which in turn determine the self-adjoint extensions of the operator in a given geometry. Self-adjoint extensions of Dirac operators in the presence of edges and domain walls are well studied. In this project, we focus on classifying these self-adjoint extensions in bounded geometries with corners, where two edges come together. Such geometries have been recently identified as an interesting platform for higher-order topological states.

**Prerequisites:** The project will combine analytical and numerical approaches as well as ideas from functional analysis and algebraic topology, but prior knowledge of these subjects is not necessary. Some knowledge of linear algebra and differential equations and experience with mathematical modeling are preferred.

**Mentor:** Chris Judge

**Title:** The space of eigendata

**Project Description:** In this project, we will explore an instance of a construction made by Karen Uhlenbeck, winner of the 2019 Abel Prize.

The construction consists of triples $(A, \lambda, v)$ where $A$ is an $n \times n$ symmetric matrix, $\lambda$ is an eigenvalue of $A$, and $v$ is an eigenvector associated to $\lambda$. Amazingly, this construction yields a 'manifold', a topological space that appears to be Euclidean in a neighborhood of any triple. We will consider basic questions about this construction that have not been considered before: How many connected components does this space have? What is the geometry of each connected component? We will also consider potential applications to the problem of 'avoided crossings' in quantum mechanics.

`https://en.wikipedia.org/wiki/Avoided_crossing.`

**Prerequisites:** A firm foundation in linear algebra and undergraduate analysis is expected. Some exposure to topological ideas would be helpful but are not necessary. Some experience with computer software such as Mathematica, Maple, Sage, and/or Matlab is expected as these should be very helpful for exploration.

**Mentor:** Elizabeth Housworth

**Title**: Evaluating Statistical Approximate Methods

**Project Description**: Taylor series expansions allow for approximate methods for calculating the mean and variance of complicated functions of simpler statistics whose mean and variances are known. This is called the delta method in mathematical statistics. In genetics, the Papazian Nonparental Ditype (NDP) ratio has been historically used to measure crossover interference. Don't worry if that last sentence doesn't make sense right now. The point is that the NDP ratio is a (somewhat) complicated function of simpler statistics whose means, variances, and covariances are well understood. The purpose of this project is to understand when two different approximate methods work well for hypothesis tests of the NDP ratio and whether one of the two methods performs consistently better than the other.

**Prerequisites:** It would be helpful if you have had programming experience in any language (the project will use R). Other essential tools for the project include multivariable calculus (partial derivatives, Taylor series expansions for function of two variables), some probability and mathematical statistics (binomial/multinomial distributions, Poisson distributions, hypothesis testing, normal distribution, etc.)

**Mentor**: Nicholas LaRacuente

**Title:** Quantum Certificates and Black Boxes

**Project Description:** A certificate is a claimed proof that a computational problem instance has a particular answer. For example, a certificate for the well-known traveling salesman problem would be a suggested path, easily checked by adding up distances. While certificates are central to the famous P vs. NP problem, there are several notions of 'quantum NP' and new questions about which of these notions are equivalent. In quantum computing, a black box or oracle is a subroutine called within a circuit or computation step. The black box may hide a complex computation or use information not otherwise available, but from the outside it appears as an elementary operation. Black box models often replace extremely hard questions about computational complexity by more tractable questions about counting and probability. The first stage of this project is a literature review. The second stage will search for opportunities to prove new results. Based on recent developments, this setting may yield opportunities for creative approaches with relatively few technical prerequisites.

**Prerequisites:** Essential: a strong linear algebra background and experience with complex numbers. Desirable: any familiarity with quantum states, probability, and/or theory of computing.

**Mentors:** Zach Babyak, Héctor Peña Pollastri and Julia Plavnik

**Title:** Generalized quantum groups associated to restricted Endymion algebras

**Project Description**: The classic mathematical structure to model symmetries is group theory, and their representations turn out to be fundamental to understand how they ap- pear in physical models. A generalization of groups are Hopf algebras, which turn out to be equivalent to groups if they are commutative or cocommutative. Because of this, it was extremely surprising when Drinfeld and Jimbo, and later Lusztig, introduced the notion of quantum groups, which are the first examples of Hopf algebras without these properties, and describe symmetries beyond group theory. These algebras are "deformations" by a parameter q of the enveloping algebras of semisimple Lie algebras. Quantum groups have attracted a lot of interest since they are at the intersection of pure mathematics and physics. In particular, they are related, for example, to Topological and Conformal Quantum Field Theories, fusion categories, Vertex Operator Algebras, solutions of the Yang-Baxter equation, and knot and link invariants. These objects have been well-studied over fields of characteristic zero but little has been explored over fields of positive characteristic. To get new examples, there is a known procedure for recovering quantum groups from a class of algebras called Nichols algebras via a braided Drinfeld double. In particular, many new finite-dimensional examples of Nichols algebras in characteristic p were introduced recently by Andruskiewitch, Angiono, and Heckenberger. We expect to construct generalized quantum groups in positive characteristic by looking into these examples. More specifically, we are interested in studying some examples arising from certain braided vector spaces called pale blocks. The corresponding algebras are called Restricted Endymion Algebras. Our main goal is to define and describe the associated quantum group and compute its simple modules. If time permits, we also propose to compute a family of indecomposable modules and fusion rules. This project will be jointly mentored by Babyak (in-person) with additional online support from Pollastri and Plavnik.

**Prerequisites:** The prerequisite for a successful project is strong linear algebra skills. Exposure to abstract algebra or computer programming may be helpful but is not required.

**Mentors:** Sanjana Agarwal and Yun Liu

**Title**: Computing $HH_0$ for polytopes in $\mathbb{R}^2$

**Project Description:** Given two polytopes in some space (imagine two triangles in $\mathbb{R}^2$ for example), one can ask the following question: is it possible to cut one of the polytopes into smaller pieces of polytopes and 'rearrange' and paste them together to form the second polytope? The available rearrangement will be determined by the chosen isometries of your space. This question was raised by Hilbert in 1900! The answer to this question depends on the dimension and the geometry of the space you choose. For two polytopes in Euclidean plane $\mathbb{R}^2$, one can do this if and only if the two polytopes have the same area (if you allow all isometries for arrangements). Now let's take all the polytopes in our space and identify the ones that are 'cut-paste-equal' to each other. Then this gives rise to a group, which we call $K_0$. There is a map from $K_0$ to another group that we are interested in. Let's call the second group $HH_0$. In this project, we will be doing some computations of group $HH_0$ when our space is $\mathbb{R}^2$ and we allow different subgroups of isometries for rearrangements.

**Prerequisites:** At least a first course in group theory or abstract algebra, at least a first course in linear algebra, at least one proof writing course (which may coincide with any of the other courses mentioned here), optional – Calculus 3