2024 REU project descriptions

Wai-Tong (Louis) Fan:

Chris Judge:

Dami Lee & Joshua Southerland:

Seppo Niemi-Colvin

Sanjana Agarwal

Shukun Wu:

Shouhong Wang:

Mentor: Wai-Tong (Louis) Fan

Title: Probabilistic models in spatial population genetics

Description: Spatial population genetics is an active research field that studies how genetic variation is distributed across space within populations and how spatial factors influence the processes of evolution, gene flow, and population structure. This field integrates principles from population genetics, spatial stochastic models and spatial statistics to explore the relationships between genetic diversity and geographic space. Research findings in this field have important applications in biodiversity conservation, disease epidemiology, and evolutionary biology. However, the underlying mathematical models are often challenging to analyze or simulate, due to their high (or infinite) dimensionality.

This undergraduate research project begins with a quick overview of existing mathematical models employed in spatial population genetics, then delves into the mathematical challenges and is complemented by applications to real life genomic data. For example, given the entire genome of n sampled individuals in the form of n DNA sequences with length L (that is, an n by L matrix whose entries are nucleotides), can we infer the migration history including the the effective population size and the migration rate of the population?

Co-requisites: Basic knowledge in random variables and Markov chains.

Mentor: Chris Judge

Title: Geometric perturbations of quantum states

Description: Imagine a single quantum particle confined to a 2-dimensional polygon. It moves around the polygon bouncing off a side when it meets one. Classical quantum theory does not predict the exact location or momentum of the particle. Instead, it provides, for example, the likelihood that the particle will lie in any given part of the polygon. Mathematically, this likelihood is the integral of a function on the polygon, the `wave function'. The purpose of this project is to understand how wave functions change as the geometry of the polygon changes. For example, what happens when the polygon is stretched and/or rotated? We will explore such questions using both thought experiments and computer experiments. In the end, we will rigorously justify our observations with proofs.

Prerequisites: The student should be fluent in undergraduate linear algebra and undergraduate analysis. No prior experience with physics is necessary though it might be helpful. Familiarity with software such as Mathematica, Maple, Matlab, or Sage is preferable.

Mentors: Dami Lee & Josh Southerland

Title: Kontsevich-Zorich monodromy groups

Description: Imagine standing at the north pole of a large sphere. Without changing the direction that you are facing, walk towards the south pole of this sphere. When you are about halfway there, walk sideways (just a little bit) along the equator, and as you do this, keep facing the south pole. Now, keeping your eye on the south pole, backstep up to the top of the sphere. Despite the fact that you never changed the direction that you were facing, you seem to be facing in a different direction than you were at the beginning. This phenomenon is called "holonomy", which is closely related to "monodromy" - how things can twist and turn on surfaces.

The primary goal of this project is to study something called the Kontsevich-Zorich monodromy groups for translation surfaces. One can think of a translation surface as an even-sided polygon, where we identify opposing, parallel edges of the same length by translation. While the question of holonomy on a translation surface is interesting (and has an answer), our focus will be on the space of translation surfaces, or moduli space, where each point in the space represents a translation surface. If we move a little bit in the moduli space, this is like slightly deforming the translation surface. It is in this moduli space where there is much active research in understanding the Kontsevich-Zorich monodromy goups - groups which are related to how closed loops on the surface evolve as we deform the surface (move in the moduli space).

In the first week of the program, we will learn what a translation surface is, and intuitively understand the moduli space of a particular type of translation surface: the torus. We will then introduce square-tiled surfaces, a special type of translation surface, and begin studying the monodromy groups associated with these surfaces. The goal of the project will be to compute monodromy groups for certain square-tiled surfaces, and attempt to answer the following question: can we find an algorithm that computes the monodromy group?

Prerequisites/Corequisites: Linear algebra, group theory, and topology (or an introduction to general topology as a part of an analysis course). We also require some experience with coding. Some knowledge of geometry and algebraic topology would be helpful, but not required.

Mentor: Seppo Niemi-Colvin

Title: Combinatorial holonomy

Description:

Rolling a sphere on a table can result in the sphere changing orientation even if one returns to exactly the same position, and this idea can be explored not only on a flat table but also on a curved surface. The ways in which these loops can change the position can be summarized in terms of the surface's holonomy group. We will explore the discrete analogue of this situation: rolling a tetrahedron over a triangulated surface, a cube over a surface made of squares, or otherwise rolling similar platonic solids over a surface made out of discrete pieces.

Co-requisites: group theory as seen in a first course in Abstract Algebra and the fundamental group from Algebraic Topology may be useful, but these concepts may also be learned as a part of the project as needed.

Mentor: Agarwal, Sanjana

Title: Understanding phyllotactic patterns via energy minimization problem on lattices

Project Description: Very broadly speaking, in this project we are interested in studying the mathematics of formation of certain shapes and patterns in nature – more particularly the Fibonacci style patterns. Phyllotaxis is the arrangement of leaves on a plant stem. The phyllotactic spirals form a distinctive class of patterns in nature. Often times these spirals contain a certain number of clockwise and counterclockwise helical formations, and these clockwise and counterclockwise helices are (upto a very small margin of error) adjacent numbers from Fibonacci sequence.

In this project we will mathematically study the formation of Fibonacci patterns via an energy minimization problem. The phyllotactic spirals can be converted to lattices. The goal is to look at the space of all lattices under certain physical constraints and analyze the energy function of this space.

Pre-requisites: Calculus, linear algebra. Basic mathematica or matlab skills will be useful along with some numerical analysis techniques. Basic familiarity with concepts of tessellations and lattices is expected but can be picked.

Mentor: Shukun Wu

Title: Sumsets of convex sequences.

Description: If A is a finite subset of the real number, its sumset, A+A, is defined by {a+b: a, b are elements of A}. There are a lot o f interesting open problems regarding the behavior of sumsets, and a significant part of them ask whether there is a full expansion on the sumset A+A when the host set A has certain properties. For example, when A is a convex sequence, i.e., A={ f(n): n=1,2,...,N, and f is a convex function}, must the size of A+A be large, in the sense that $|A+A| > K_c |A|^{2-c}$ is true for any $c>0$? We are going to investigate the sumsets for convex sequences in the REU project, partly by looking into some examples.

Prerequisite: College-level calculus, linear algebra. Familiarity with Fourier analysis would be beneficial, but it is not required.

Mentor: Shouhong, Wang

Title: Dynamic transitions and Pattern Formations in Bacterial Chemotaxis

Description:

In this project we will explore dynamic transitions and pattern formations of bacterial chemotactic systems, including in particular the Keller-Segel Model. The prerequisites include undergraduate level Calculus 1-3, linear algebra and differential equations.