

Cutting and pasting of manifolds and group actions

Suppose M is a closed, smooth, oriented manifold which can be cut open along a codimension 1 submanifold. By doing this cutting operation we obtain two manifolds with the same boundary which we can now glue back together using an automorphism of the boundary which is not the identity. The new object that we obtain is said to be “cut and paste” or SK -equivalent to the manifold M we started with. Here is an example of a cut and paste operation:

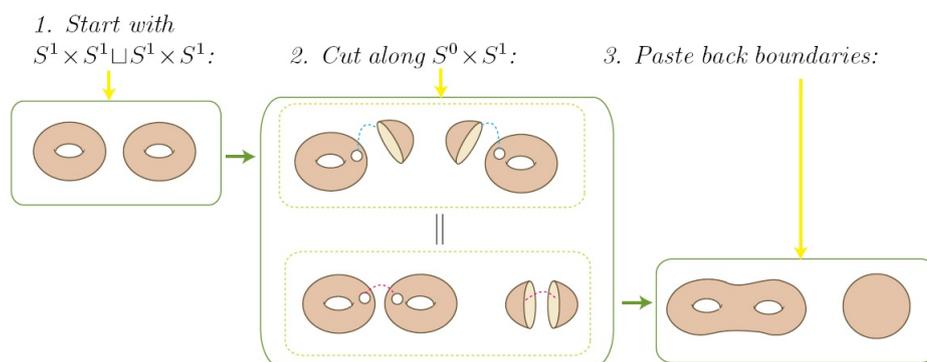


Figure 1: Example of cut and paste operation

It turns out that the cut and paste operation doesn't only define an equivalence relation, it defines certain groups called the SK -groups. The SK -groups were first defined some 40 years ago, but they haven't been developed or investigated in depth since then, even though they have interesting connections with very active areas of research.

An interesting topic is to understand which topological invariants are invariants of the cut and paste operation. That is, if we evaluate a certain invariant like the Euler characteristic or the signature on two cut and paste equivalent manifolds do we obtain the same result?

Another interesting topic is to investigate equivariant versions of the SK -groups where a group action is involved. For example, if we think of an interval, we can have a \mathbb{Z}_2 -action (with generator t) which will swap both sides of the interval and leave the middle point fixed:



Figure 2: Interval $[-1, 1]$ and interval with \mathbb{Z}_2 -action

How can we define cut and paste operations when there is such a group action involved?

We will attempt to tackle these questions and give computations of these groups which are missing in the Literature.

Depending on the background of the student we will adjust the goals for the summer project, but in any case this project will provide the student with a first insight into an exciting area of research and allow for further work in the area.

Prerequisites: Some familiarity with basic algebraic topology would be desirable.